Name: ______________________________________________ Period: __________ Date: _______________

# Solving Radical Equations Guide Notes

**A radical equation** is an equation that contains a radical expression with a variable in the radicand.

**Steps to Solve an Equation Containing One Square Root Term**

1. Isolate the radical term.
2. Square both sides of the equation.
3. Solve the resulting equation.
4. Check your solution. Watch for extraneous solutions.

A proposed solution that is not a solution of the original equation it is called **an extraneous solution**. Extraneous solutions are false solutions and do not satisfy the original equation.

Radical equations with square roots often have extraneous solutions because through the process of solving these equations we must square both sides of the equation. However, the process of squaring both sides is not a "reversible" operation.

Sample Problem 1: Solve the following equation.

**a.**

$$\sqrt{x-1}=2$$
$$\left(\sqrt{x-1}\right)^2=2^2$$
$$x-1=4$$
$$x=5$$

Checking solution:

$$x=5$$
$$\sqrt{5-1}=2$$
$$\sqrt{4}=2$$
$$2=2$$

$x=5$ is a solution of this equation

$\{5\}$

**b.**

$$\sqrt{6x}+6=0$$
$$\sqrt{6x}=-6$$
$$\left(\sqrt{6x}\right)^2=(-6)^2$$
$$6x=36$$
$$x=6$$

Checking solution:

$$x=6$$
$$\sqrt{6*6}+6=0$$
$$\sqrt{36}=-6$$
$$6\neq-6$$

$x=6$ is an extraneous solution of this equation

$\emptyset$

**c.**

$$2\sqrt{x-6}-3=5$$
$$2\sqrt{x-6}=3+5$$
$$2\sqrt{x-6}=8$$
$$\left(2\sqrt{x-6}\right)^2=8^2$$
$$4(x-6)=64$$
$$4x-24=64$$
$$4x=24+64$$
$$4x=88$$
$$x=22$$

Checking solution:

$$x=22$$
$$2\sqrt{22-6}-3=5$$
$$2\sqrt{16}-3=5$$
$$2\sqrt{16}-3=5$$
$$2*4-3=5$$
$$8-3=5$$
$$5=5$$

$x=22$ is a solution of this equation

$\{22\}$

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# Solving Radical Equations Guide Notes

**d.**

$\sqrt{6x+1} - x = -1$
$\sqrt{6x+1} = x - 1$
$\left(\sqrt{6x+1}\right)^2 = (x-1)^2$
$6x + 1 = x^2 - 2x + 1$
$8x - x^2 = 0$
$x(8 - x) = 0$
$x_1 = 0$

$x_2 = 8$

Checking solution:
$x_1 = 0$ .
$\sqrt{6x+1} - x = -1$
$\sqrt{6 * 0 + 1} - 0 = -1$
$\sqrt{1} \neq -1$
$x_1 = 0$ is an extraneous solution of this equation

$x_2 = 8$
$\sqrt{6 * 8 + 1} - 8 = -1$
$\sqrt{49} - 8 = -1$
$7 = 7$
$x_2 = 8$ is a solution of this equation
$\{8\}$

**Steps to Solve an Equation Containing Two Square Root Terms**

1. Isolate one of the radical terms
2. Square both sides of the equation.
3. Isolate the remaining radical term.
4. Square both sides of the equation.
5. Solve the resulting equation.
6. Check your solution. Watch for extraneous (false) solutions.

**Sample Problem 2**: Solve the following equation.

**a.**

$\sqrt{5x+3} = \sqrt{3x+7}$
$\left(\sqrt{5x+3}\right)^2 = \left(\sqrt{3x+7}\right)^2$
$5x + 3 = 3x + 7$
$x = 2$

Checking solution:
$x = 2$
$\sqrt{5 * 2 + 3} = \sqrt{3 * 2 + 7}$
$\sqrt{10 + 3} = \sqrt{6 + 7}$
$\sqrt{13} = \sqrt{13}$
$x = 2$ is a solution of this equation
$\{2\}$

**b.**

$\sqrt{2x-3} - \sqrt{x+2} = 0$
$\sqrt{2x-3} = \sqrt{x+2}$
$\left(\sqrt{2x-3}\right)^2 = \left(\sqrt{x+2}\right)^2$
$2x - 3 = x + 2$
$2x - x = 3 + 2$
$x = 5$

Checking solution:
$\sqrt{2x-3} - \sqrt{x+2} = 0$
$\sqrt{2 * 5 - 3} - \sqrt{5 + 2} = 0$
$\sqrt{7} - \sqrt{7} = 0$
$\sqrt{7} = \sqrt{7}$
$x = 5$ is a solution of this equation
$\{5\}$



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